AQA

A-LEVEL
Mathematics

Mechanics 3 – MM03

Mark scheme

6360

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Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1 (a)	$x = 4\sqrt{3}t$	B1		
	$y = 4t - \frac{1}{2}gt^2$	B1		
	$t = \frac{x}{4\sqrt{3}}$			
	$y = 4 \times \frac{x}{4\sqrt{3}} - \frac{1}{2}(9.8)\left(\frac{x}{4\sqrt{3}}\right)^2$	M1		
	$y = \frac{x}{\sqrt{3}} - \frac{49x^2}{480}$	A1	4	AG
(b)	$y = \frac{4}{\sqrt{3}} - \frac{49(4)^2}{480}$	M1		PI by correct answer
	(The height is $0.676 + 0.3$) 0.98 m or 98 cm	A1	2	CAO
(c)	No air resistance or The ball does not spin **or** No loss of energy	B1	1	
	Total		**7**	

Q	Solution	Mark	Total	Comment
2	$[J] \equiv \mathrm{MLT^{-1}}$ $[g] \equiv \mathrm{LT^{-2}}$	B1		Dimensions of J and g, PI
	$\mathrm{MLT^{-1}} = L^{\alpha}(\mathrm{ML^2})^{\beta}(\mathrm{LT^{-2}})^{\gamma}$	M1		FT from B1
	$\mathrm{MLT^{-1}} = \mathrm{M}^{\beta}\mathrm{L}^{\alpha+2\beta+\gamma}\mathrm{T}^{-2\gamma}$	A1		PI
	$\beta = 1$	B1		
	$-2\gamma = -1$ $\alpha + 2\beta + \gamma = 1$	m1		Correctly solving **their** two equations **involving three unknowns,** PI by the answers
	$\gamma = \frac{1}{2}$ $\alpha = -\frac{3}{2}$	A1		
	Total		**6**	

(a) Only quoting the formula and substituting scores M1 A1.

Q	Solution	Mark	Total	Comment
3 (a)	$I = \int_0^3 (3t+1)\,dt$	M1		Condone missing limits and missing dt
	$= \left[\frac{3}{2}t^2 + t\right]_0^3$	m1		For correct integration only
	$= \frac{33}{2}$ or 16.5 Ns	A1	3	Condone missing units
(b)	$\frac{33}{2} = 0.5v - 0.5(4)$	M1		Impulse/momentum equation for correct terms, FT on their impulse from part (a)
	$v = 37$ ms^{-1}	A1F	2	
(c)	$\int_0^T (3t+1)\,dt = 0.5(20) - 0.5(4)$ $\left[\frac{3}{2}t^2 + t\right]_0^T = 0.5(20) - 0.5(4)$	M1		Correct impulse-momentum equation, condone missing limits
	$3T^2 + 2T - 16 = 0$	A1		Correct quadratic equation
	$(3T+8)(T-2) = 0$ or $T = \frac{-2 \pm \sqrt{(-2)^2 - 4(3)(-16)}}{2(3)}$	m1		Correct solution of their equation, PI
	$T = 2$ s $\left(T = -\frac{8}{3}\text{ s impossible}\right)$	A1	4	Rejecting impossible time PI
	Total		**9**	

(a)
Alternative (non-calculus): Attempt at finding the area under force-time graph M1

$= \frac{1+10}{2} \times 3$ OE A1

$= 33/2$ or 16.5 (NS) A1

(c)
Alternative:

$a = \frac{3t+1}{0.5}$

$v = \int \frac{3t+1}{0.5}(dt)$ Attempt at integrating the acceleration M1

$v = 3t^2 + 2t + 4$

$20 = 3T^2 + 2T + 4$

$3T^2 + 2T - 16 = 0$ A1 , etc.

Alternative (non-calculus): Attempt at finding the area under force-time graph for impulse

$$\frac{1+(3T+1)}{2}\times T = 0.5(20) - 0.5(4)$$ OE M1

Q	Solution	Mark	Total	Comment
4 (a)	$\boldsymbol{v}_A = \dfrac{(-\mathbf{i}+3\mathbf{j})-(\mathbf{i}+2\mathbf{j})}{\frac{1}{2}} = -4\mathbf{i}+2\mathbf{j}$ $\boldsymbol{v}_B = \dfrac{(2\mathbf{i}-\mathbf{j})-(-\mathbf{i}+\mathbf{j})}{\frac{1}{2}} = 6\mathbf{i}-4\mathbf{j}$	M1 A1		M1 for a difference of two corresponding position vectors divided by $\frac{1}{2}$, A1 for all correct
	${}_A\boldsymbol{v}_B = (-4\mathbf{i}+2\mathbf{j})-(6\mathbf{i}-4\mathbf{j})$	m1		Accept any difference of their velocities
	${}_A\boldsymbol{v}_B = -10\,\mathbf{i}+6\mathbf{j}$	A1	4	NMS scores 0/4
(b)	$\mathbf{r_0} = (\mathbf{i}+2\mathbf{j})-(-\mathbf{i}+\mathbf{j})$	B1		
	$\mathbf{r} = (\mathbf{i}+2\mathbf{j})-(-\mathbf{i}+\mathbf{j})+(-10\mathbf{i}+6\mathbf{j})t$	M1		M1 for using $\mathbf{r} = \mathbf{r}_0 + {}_A\boldsymbol{v}_B \boldsymbol{t}$ with their ${}_A\boldsymbol{v}_B$.
	$\mathbf{r} = (2-10t)\mathbf{i}+(1+6t)\mathbf{j}$	A1	3	AG
(c)	$AB^2 = (2-10t)^2+(1+6t)^2$	M1		Or $AB = \sqrt{(2-10t)^2+(1+6t)^2}$
	A and B are closest when $\dfrac{\mathrm{d}AB^2}{\mathrm{d}t}\left(\text{or } \dfrac{\mathrm{d}AB}{\mathrm{d}t}\right) = 0$	B1		PI
	$\dfrac{\mathrm{d}AB^2}{\mathrm{d}t} = 2(2-10t)(-10)+2(1+6t)6 = 0$	m1 A1		Condone one sign error **or** one coefficient error All correct
	$t = \dfrac{7}{68}$ or 0.103	A1	5	At least 3 s.f. required if decimal. Accept equivalent fractions
(d)	$AB = \sqrt{(2-10\times 0.103)^2+(1+6\times 0.103)^2}$ **or** $\sqrt{\left(\frac{33}{34}\right)^2+\left(\frac{55}{34}\right)^2}$	m1		Dependent on M1 and m1 in part (c)
	$AB = 1.89$ or $1.886...$	A1	2	At least 3 s.f. required
	Total		**14**	

4 (c) *Alternative 1*:

$AB^2 = (2-10t)^2 + (1+6t)^2$ M1

$AB^2 = 4 - 40t + 100t^2 + 1 + 12t + 36t^2$ A1

A and B are closest when $\frac{\mathrm{d}AB^2}{\mathrm{d}t}\left(\text{or } \frac{\mathrm{d}AB}{\mathrm{d}t}\right) = 0$ B1

$-40 + 200t + 12 + 72t = 0$ m1

$t = \frac{7}{68}$ or 0.103 A1

4 (c) *Alternative 2*:

$AB^2 = (2-10t)^2 + (1+6t)^2$ M1

$AB^2 = 4 - 40t + 100t^2 + 1 + 12t + 36t^2$ A1

$AB^2 = 136t^2 - 28t + 5$

$AB^2 = 136\left(\left(t - \frac{7}{68}\right)^2 + \ldots\right)$ m1 A1 m1 for attempt at completing the square of <u>their</u> quadratic

$t = \frac{7}{68}$ or 0.103 A1

4(c) *Alternative 3 (Not in the specification)*:

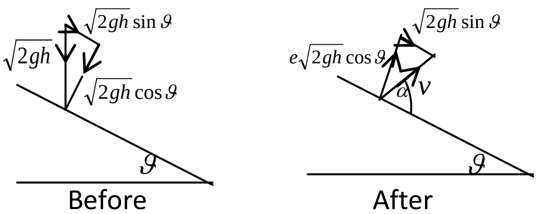
$[(2-10t)\mathbf{i} + (1+6t)\mathbf{j}].[-10\mathbf{i} + 6\mathbf{j}]\ (=0)$ M1 for the scalar product of the r with <u>their</u> ${}_A v_B$

A1 for all correct

$-20 + 100t + 6 + 36t \quad (=0)$ A1

$-20 + 100t + 6 + 36t = 0$ m1 for correctly solving <u>their</u> equation

$t = \frac{7}{68}$ or 0.103 A1

Q	Solution	Mark	Total	Comment
5 (a)	‘No change’ with an attempt to explain	B1		
	Explanation referring to smoothness or lack of friction parallel to the plane	B1	2	
(b)				
	Speed before impact $= \sqrt{2gh}$ PI	M1		Allow $\pm$ expressions
		A1		
	Parrallel component after impact $= \sqrt{2gh}\sin\vartheta$	A1		
	Perpendicular component after impact $= e\sqrt{2gh}\cos\vartheta$		3	
(c)		M1 A1		Allow M1 for using $\sin\vartheta$ instead of $\cos\vartheta^*$ and + instead of –
	At B,) $0 = e\sqrt{2gh}\cos\vartheta^* \, t - \frac{1}{2}g\cos\vartheta\, t^2$			
		A1		
	$t = \frac{2e\sqrt{2gh}\cos\vartheta}{g\cos\vartheta}$ or $\frac{2e\sqrt{2gh}}{g}$	M1 A1		Allow M1 for using $\cos\vartheta$ instead of $\sin\vartheta^*$ and – instead of +
	$x = \sqrt{2gh}\sin\vartheta^* \, t + \frac{1}{2}g\sin\vartheta\, t^2$			
		m1		Elimination of t. OE
	$AB = \frac{\sqrt{2gh}\sin\vartheta 2e\sqrt{2gh}}{g} + \frac{g\sin\vartheta 4e^2 2gh}{2g^2}$			
	$AB = \frac{4ghe\sin\vartheta}{g} + \frac{8g^2he^2\sin\vartheta}{2g^2}$	A1	7	AG, must be convinced
	$AB = 4he\sin\vartheta + 4he^2\sin\vartheta$			
	$AB = 4he(e+1)\sin\vartheta$			
	Total		**12**	

(a) The minimum statement for 2 marks is: ‘No friction, so no change to velocity parallel to the plane’

Allow numerical value of 9.8 for g in part (c), but deduct one A1 mark in part (b) if they have used numerical value.

5(c) *Alternative*

(At B,) $0 = v\sin\alpha\, t - \frac{1}{2}gt^2\cos\vartheta$	M1
$t = \dfrac{2v\sin\alpha}{g\cos\vartheta}$	m1
$x = v\cos\alpha t + \frac{1}{2}gt^2\sin\vartheta$	M1
$AB = v\cos\alpha\left(\dfrac{2v\sin\alpha}{g\cos\vartheta}\right) + \dfrac{1}{2}g\left(\dfrac{2v\sin\alpha}{g\cos\vartheta}\right)^2\sin\vartheta$	A1
$AB = \dfrac{2v^2\sin\alpha\cos\alpha}{g\cos\vartheta} + \dfrac{2v^2\sin^2\alpha\sin\vartheta}{g\cos^2\vartheta}$	
$\sin\alpha = \dfrac{\sqrt{2gh}\,e\cos\vartheta}{v}$, $\cos\alpha = \dfrac{\sqrt{2gh}\sin\vartheta}{v}$	B1 (for both)
$AB = \dfrac{2v^2 \times \dfrac{\sqrt{2gh}\,e\cos\vartheta}{v} \times \dfrac{\sqrt{2gh}\sin\vartheta}{v}}{g\cos\vartheta} + \dfrac{2v^2\left(\dfrac{\sqrt{2gh}\,e\cos\vartheta}{v}\right)^2\sin\vartheta}{g\cos^2\vartheta}$	m1
$AB = 4he\sin\vartheta + 4he^2\sin\vartheta$	
$AB = 4he(e+1)\sin\vartheta$	A1 AG, must be convinced

Q	Solution	Mark	Total	Comment
6 (a)	Conservation of linear momentum along the line of centres:			
	$2\times 3\cos 60^\circ - 4\times 5\cos 60^\circ = 2\times v$	M1		Condone sign errors
		A1		Correct with $2v$ or $-2v$
	$v = -3.5$	A1		Or $\frac{7}{2}$, accept 3.5 from consistent working
	Velocity of $A \perp$ to line of centres: $3\sin 60^\circ$	B1		Possibly seen on a diagram
	$V = \sqrt{(3.5)^2 + (3\sin 60^\circ)^2}$	M1		FT their v from above
	$V = 4.36$ or $\sqrt{19}$ ms^{-1}	A1	6	AWRT 4.36, condone missing units
(b)	$\tan^{-1}\frac{3\sin 60^\circ}{3.5}$ *	M1		For correct expression, FT their v from part (a)
	$= 37^\circ$	A1	2	CAO
(c)	$e = \frac{3.5}{3\cos 60^\circ + 5\cos 60^\circ}$	M1		For correct expression, FT their v from part (a)
	$e = 0.875$ or $\frac{7}{8}$	A1	2	CAO
(d)	$I = 4\times 5\cos 60^\circ - 4\times 0$ or $2\times 3\cos 60^\circ - -2\times 3.5$	M1		OE, condone the missing zero term, **FT**
	$I = 10$ Ns	A1	2	CAO, condone missing units
	Total		**12**	

(b)	* or $\sin^{-1}\frac{3\sin 60^\circ}{4.36}$ or $\cos^{-1}\frac{3.5}{4.36}$

Q	Solution	Mark	Total	Comment
7 (a)	$J = 2m(2u) - 2m(0)$	M1		
	$= 4mu$	A1	2	A0 for sign error or $-4mu$ as answer
(b)	$2m(2u) = 2mv_A + mv_B$ $4u = 2v_A + v_B$	M1		CLM
	$\frac{v_B - v_A}{2u - 0} = \frac{2}{3}$	M1		Restitution, condone sign error
	$4u = 3v_B - 3v_A$	A1		All correct
	$v_A = \frac{8}{9}u$	A1		
	$v_B = \frac{20}{9}u$	A1	5	
(c)	$t = \frac{s-r}{\frac{20u}{9}}$ or $\frac{9(s-r)}{20u}$	M1		$(s-r)$ divided by their v_B from (b)
	Distance travelled by A is $\frac{8u}{9} \times \frac{9(s-r)}{20u}$	m1		Their $v_A \times$ their time from the line above
	$= \frac{2(s-r)}{5}$	A1		OE
	Distance of centre of A from the wall is $s + 2r - \frac{2(s-r)}{5} = \frac{3s+12r}{5}$	A1	4	AG
(d)	$w_B = \frac{20u}{9} \times \frac{2}{5}$	M1		Their v_B from (b) $\times \frac{2}{5}$
	$= \frac{8}{9}u$	A1		
	A and B have the same speed $\Rightarrow$ The distance between them will be halved to $\frac{1}{2}\left(\frac{3s+12r}{5} - 3r\right)$ or $\frac{3s-3r}{10}$	M1		Explanation not needed
	$\therefore$ The required distance is $\frac{1}{2}\left(\frac{3s+12r}{5} - 3r\right) + r = \frac{3s+7r}{10}$	A1	4	Simplification not required
	Total		**15**	

(a) Condone omission of $-2m(0)$.

7(d) *Alternative1*:

$w_B = \frac{20u}{9} \times \frac{2}{5}$ M1

$= \frac{8}{9}u$ A1

Time taken by B to collide again $= \dfrac{x}{\frac{8}{9}u}$

Time taken by A to collide again $= \dfrac{\frac{3s+12r}{5} - 3r - x}{\frac{8}{9}u}$

$x = \frac{3s+12r}{5} - 3r - x$ or $\frac{3s-3r}{10}$

The distance of the centre of B from the wall $= \frac{3s-3r}{10} + r = \frac{3s+7r}{10}$ A1

Alternative 2:

$w_B = \frac{20u}{9} \times \frac{2}{5}$ M1

$= \frac{8}{9}u$ A1

Velocity of A relative to B = $\frac{16u}{9}$

Distance to collision $= \frac{3s+12r}{5} - 3r$

Time to collision $= \dfrac{\frac{3s+12r}{5} - 3r}{\frac{16u}{9}}$

$= \frac{27s-27r}{80u}$

Distance moved by B = $\frac{8u}{9}\left(\frac{27s-27r}{80u}\right)$ M1

The required distance $= \frac{8u}{9}\left(\frac{27s-27r}{80u}\right) + r = \frac{3s+7r}{10}$ A1